RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

SECOND YEAR [2014-17]

B.A./B.Sc. THIRD SEMESTER (July – December) 2015

Mid-Semester Examination, September 2015

Date : 16/09/2015

MATH FOR ECO (General)

Time : 12 noon – 1 pm

Paper : III

Full Marks : 25

Group – A

Answer **any four** : [4 × 3]

1. Show that all circles of vadius r are represented by the differential equation $\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2} = r\frac{d^2y}{dx^2}$.

2. Solve : $(y^4 - 2x^3y)dx + (x^4 - 2xy^3)dy = 0$.

3. Solve : $\frac{dy}{dx} = \frac{y-x+1}{y+x+5}$.

4. Solve : $(\cos y + y\cos x)dx + (\sin x - x\sin y)dy = 0$.

5. Solve : $y(2xy + e^x)dx - e^x dy = 0$.

6. Solve : $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$.

7. Solve : $ayp^2 + (2x - b)p - y = 0,\ a > 0$.

Group – B

Answer **any two** : [2 × 2·5]

8. State implicit function theorem for a function of two veriables. [2.5]

9. a) Prove that the function $f(x,y) = \begin{cases} \frac{x^3+y^3}{x-y} & , \ x \neq y \\ 0 & , \ x = y \end{cases}$

   is not continuous at (0,0).

   b) What is the set of all interior points of $\mathbb{R}$ if we consider $\mathbb{R}$ as a subset of $\mathbb{R}^2$? [2 + 0·5]

10. If $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi$ and $z = r\cos\theta$ then show that $\frac{\partial x}{\partial r}.\frac{\partial r}{\partial x} + \frac{\partial x}{\partial \theta}.\frac{\partial \theta}{\partial x} + \frac{\partial x}{\partial \phi}.\frac{\partial \phi}{\partial x} = 1$. [2.5]

11. By $\in - \delta$ definition show that $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{xy(x^2-y^2)}{x^2+y^2} = 0$. [2.5]

Answer **any two** : [2×4]

12. If $f(x,y) = \begin{cases} xy\frac{x^2-y^2}{x^2+y^2} & , \ (x,y) \neq (0,0) \\ 0 & , \ (x,y) = (0,0) \end{cases}$

    then show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

13. State and prove converse of Euler theorem for a homogenous function of three variables.

14. Show that the function, $f(x,y)=\begin{cases}\dfrac{xy}{\sqrt{x^2+y^2}} & , \quad (x,y)\neq(0,0)\\ 0 & , \quad (x,y)=(0,0)\end{cases}$

is not differentiable at (0,0) although f is continuous at (0,0).

15. If α,β,γ are the roots of the equation $f(t)=\dfrac{u}{a+t}+\dfrac{v}{b+t}+\dfrac{w}{c+t}-1=0$

then show that $\dfrac{\partial(u,v,w)}{\partial(\alpha,\beta,\gamma)}=-\dfrac{(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)}{(a-b)(b-c)(c-a)}$.

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